Comments on "Line Width and Bandwidth of Millimeter-Wave Resonance Isolators"*

I. Bady

The writer cannot agree with some of the principal conclusions in the article by P. Vilmur and K. Ishii.[1] The authors report a very narrow line width for a single crystal barium ferrite slab. The line width is very frequency sensitive, ranging from 3.5 oersteds at 58 Gc to 13 oersteds at 59 Gc. The authors derive a formula for line width which they claim fits the experimental data. However, it is believed that the narrow line width is not necessarily an intrinsic property of the material, but is influenced by the relatively large thickness of the slab used. Also this writer does not agree with the formula for line width derived by the authors.

Let us consider first the formula for line width derived by Vilmur and Ishii. The following is an outline of the procedure used by the authors.

1) Eq. (7) of the article can be taken as the starting point. It expresses the attenuation due to a ferrite slab as a function of geometry and material properties.

2) The authors then make some approximations and then rewrite (7) in a new form, in which all terms with the same power of ω are gathered together. The result is (13). Eqs. (7) and (13) are clearly equivalent and differ only in the arrangement of the terms and in the fact that (13) contains approximations not included in (7). Some comments will be made later concerning one of the approximations.

3) Going back to (7), the authors determine the maximum value of the attenuation by setting $\omega=\omega_r$. Dividing the maximum attenuation by 2 gives the "3 db points." This is expressed in (17).

4) The authors then substitute this value of half the maximum attenuation into (13), make some additional approximations and obtain (21), a quadratic in ω^2. Solving for ω^2 yields two values, ω_1^2 and ω_2^2. From these values formulas for bandwidth and line width are obtained.

This writer feels that the above procedure is a very complicated and, as it turns out, erroneous way to obtain the formula for line width. The method that is generally used to obtain a formula for line width in terms of material parameters is much simpler. Consider (7). Each of the terms on the right-hand side have the same denominator, and this denominator varies rapidly with frequency in the vicinity of resonance. The numerators, however, vary slowly with frequency. At resonance, the value of the denominator is $4\omega_r^2$. In order to determine the value of ω at which the attenuation is half the value at resonance, we need simply determine the value of ω for which the denominator is double its value at resonance. This amounts to setting $T(\omega_r^2-\omega^2-1/T^2)=\pm 2\omega_r$ and solving for ω.

This is, of course, an approximation. However, in view of the very fast variation of the denominator with frequency, and the very slow variation of the numerators with frequency, this is an accurate approximation.

Neglecting the term $1/T^2$ in the parenthesis, we obtain

$$T(\omega_r+\omega)(\omega_r-\omega_1) \approx 2T\omega_r(\omega_r-\omega_1) = 2\omega_r$$

$$T(\omega_r+\omega)(\omega_2-\omega_r) \approx 2T\omega_r(\omega_2-\omega_r) = 2\omega_r$$

$$\omega_2-\omega_1 = \Delta\omega = \frac{2}{T}.$$

This simple formula for $\Delta\omega$ should be compared to (27) in the paper.

Since $T=1/\alpha\omega_r$ and $\Delta\omega=\gamma\Delta H$ we have,

$$\Delta H = \frac{2\alpha\omega_r}{\gamma}.$$

Vilmur and Ishii give the above formula in (39). However, they dismiss it, claiming it does not fit the experimental data. The formula for line width which the authors have derived using the procedure described above is given in (38). It differs radically from (39). The authors claim that (38) fits the experimental data. However, this is accomplished simply by the expedient of adjusting seven constants, C_1 to C_7, to fit experimental curves.

This writer believes that (39) is correct and (38) is incorrect. There does not appear to be any advantage in the complicated procedure used by the authors, over the direct procedure described above. On the other hand, the numerous approximations made by Vilmur and Ishii, in order to keep the process manageable, weaken their procedure. One approximation used by the authors that this writer feels to be particularly poor is in the improper use of the approximation $T^2\omega_r^2\gg 1$ in deriving expression (9) from expression (8). A more accurate form of expression (9) is shown below.

$$\frac{\alpha}{C\omega_m T^3}\left[T^4\omega^4-2\omega_r^2\omega^2T^4+\omega_r^4T^4+4\omega_r^2T^2\right]. \quad (9')$$

Expression (9′) differs from expression (9) in that it includes the term $4\omega_r^2T^2$ which the authors omitted in their approximation. Far from resonance, the omission of this term does not matter. However, in the *vicinity* of resonance, particularly at the frequency where the attenuation is half its maximum value, omission of this term results in a significant error.

The authors comment that the formula for line width given in (39) does not fit the experimental data. However, this is not surprising in view of the relatively large thickness of the slab. Vilmur and Ishii's procedure in deriving (38) and this writer's procedure in deriving (39) are both based on (7), and this equation is valid only when the thickness of the slab is small enough for perturbation theory to be valid. The slab used in the authors' experiments was 0.0117 inch thick. Taking the relative dielectric constant as 16 (barium ferrite has a larger dielectric constant than spinels), and taking the permeability as 1, the electrical thickness of the slab at 59 Gc is 84.5 degrees. This surely does not warrant the use of perturbation theory.

The behavior of the single crystal barium ferrite slab, as reported by the authors, is indeed anomolous. A somewhat similar anomolous behavior has been noted by this writer[2] in connection with planar ferrites, where it was found that the apparent line width of a slab was a function of the slab thickness. The reason for the anomolous behavior of the slabs as noted by Vilmur and Ishii, and by this writer, warrants investigation. It is believed, however, that the explanation offered by Vilmur and Ishii is incorrect.

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P. Vilmur and K. Ishii[3]

Bady's main point seems to be that his simple derivation for bandwidth tends to invalidate the more complicated expression derived by the authors with the "errors" in the derivation lending support to his claim. The authors do not agree with this contention. The reason for going through the complicated steps outlined by Bady was to obtain relationships between bandwidth and resonant frequency and line width and resonant frequency because experimental data showed that such relationships existed. Obviously, Bady's derivation does not lead to any such results. Bady claims his simple formula for the bandwidth $\Delta\omega=2/T$ is exact and the authors' complicated bandwidth equation (27) is an error. The author is going to show that Bady's simple equation $\Delta\omega=2/T$ will not explain the experimental results. The value of the phenomenological relaxation time T can be estimated using an equation derived by B. Lax.[4]

$$T \approx \sqrt{R}/2\omega_r.$$

The authors' experimental results showed that, for a single crystal barium ferrite isolator, the maximum value of the reverse-to-forward loss

$$R = 30 \text{ db} = 1000$$

at the resonance frequency

$$f_r = 58.6\text{k Mc.}$$

That is to say,

$$\omega_r = 2\pi f_r = 2\pi \times 58.6 \times 10^9$$
$$= 3.68 \times 10^{11} \text{ rad/sec;}$$

substituting the experimentally obtained values of R and ω_r into Lax's approximate formula,

$$T \approx \frac{\sqrt{R}}{2\omega_r} = \frac{\sqrt{1000}}{2\times 3.68\times 10^{11}}$$
$$= 4.3 \times 10^{-11} \text{ sec;}$$

* Received February 25, 1963.

[1] P. Vilmur and K. Ishii, "Line width and bandwidth of millimeter-wave resonance isolators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 108–113, March, 1962.

[2] I. Bady, "Frequency doubling using ferrite slabs, particularly planar ferrites," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 55–64; January, 1962.

[3] Received March 25, 1963.

[4] B. Lax, "Frequency and loss characteristics of microwave ferrite devices," Proc. IRE, vol. 44, pp. 1368–1385; October, 1956.

substituting Bady's simple formula

$$\Delta\omega = \frac{2}{T} = \frac{2}{4.3 \times 10^{-11}} = 4.65 \times 10^{10}.$$

Therefore, according to Bady, the bandwidth must be

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{4.65 \times 10^{10}}{2\pi} = 7.35 \times 10^{9} = 7350 \text{ mc!}$$

The authors have never seen such a wide-band *single crystal* barium ferrite isolator. In fact, as shown in Fig. 2 of the authors' paper, the authors' experimental results showed that

$$\Delta f = 60 \text{ Mc}.$$

Bady's simplified equation $\Delta\omega = 2/T$ failed to explain the experimental results. The experimental value of $\Delta f = 60$ Mc was obtained by using Demornay Bonardi Model B-715-1, Ser. 2217, precision cavity frequency meter and the accuracy of the cavity frequency meter at 58.6 kMc is ± 1 Mc. Therefore, if it were really $\Delta f = 7350$ Mc as Bady's equation predicted, it would have been identified clearly by the precision cavity wavemeter. The fact was, however, $\Delta f = 60$ Mc and according to the authors' "complicated" equation (27), as shown in Fig. 2 of the paper, $\Delta f = 55$ Mc. Thus, the authors' (27) predicts the value of the bandwidth far better than the Bady's simple equation.

Using this simple equation and $T = 1/\alpha\omega_r$ and $\Delta\omega = \gamma\Delta H$ which the authors think unusable in this bounded system, Bady derived a simple equation of line width

$$\Delta H = \frac{2\alpha\omega_r}{\gamma}.$$

Bady claims that the above equation is correct and the authors' (38) is incorrect. If Bady were right,

$$\Delta H = \frac{2\alpha\omega_r}{\gamma} = \frac{\Delta\omega}{\gamma}$$

and γ is the gyromagnetic ratio[5,6] $= 2.8$ Mc/oe, substituting the numerical values obtained into Bady's equation,

$$\Delta H = \frac{\Delta\omega}{\gamma} = \frac{7350 \text{ Mc}}{2.8 \text{ Mc/oe}} = 2630 \text{ oersted!}$$

The authors have never seen a single crystal barium ferrite with so wide a line width. As shown in Fig. 3 of the authors' paper, the experimental results indicated that $\Delta H = 8.5$ oersted.

Bady's equation failed again to predict the line width. The experimental line width was obtained by using a calibrated micrometer-driven precision-variable magnet which accuracy was ± 1 oersted at 5000 oersteds. The precision-variable magnet was calibrated using Dyna Empire Model D-79, Ser. 907 gaussmeter. Therefore, if Bady's equation were correct, it would have been evident. The fact was opposite and the line-width was only $\Delta H = 8.5$ oe. The authors' "complicated" equation (38) predicted $\Delta H = 7.2$ oe at 58.6 kMc as shown in Fig. 3 of the paper. Thus, (38) is close to correct and Bady's equation is incorrect to use in this particular case. Bady's equations *might* be correct for unbounded ferrite but certainly not for this case. This is a bounded system. In other words, the authors feel that this simple relation for bandwidth or line width derived by Bady has meaning only for an unbounded ferrite medium. One reason for this claim is that the relation $\Delta\omega = \gamma\Delta H$ which Bady used represents an internal unmeasurable line width in this method. The externally measured line width in this method must take into account the ferrite geometry, the coupling between waveguide and the ferrite, and the anistropy field. In (28) through (38) of the authors' paper an equation for line width is developed which takes into account these factors.

An important thing to make clear is the definition of the "line width." The authors discussed it from the design engineer's viewpoint. The line width appeared through the authors' paper, is the "external" line width which is "actually measurable" quantity. The external line width is the line width of the bounded ferrite system, in which the coupling between ferrite and waveguide is taken into consideration.

The accuracy of the whole paper has been questioned, in particular the validity of dropping the term $4\omega_r^2T^2$ from expression (9). The fact still remains that, if this term is retained as in expression (9′) of Bady's communication, the term would have been dropped anyway later in the derivation as being negligible *at or near resonance*. This is exactly where Bady claims the error occurs in dropping this term. Carrying through with the derivation using expression (9′) (16) would be

$$K_3 = \frac{\alpha T}{C\omega_m}\omega_r^4$$

$$+ \frac{4\alpha}{C\omega_m T}\omega_r^2 + S_1A^0k - S_2A^1 + \frac{S_3k^2\omega_r}{\sqrt{\epsilon_0\mu_0}} \cdot \quad (16')$$

Comparing the second term (the one originally dropped) with the first term, it is seen that they differ by the factor $4/T^2\omega_r^2$. The magnitude of this factor can be estimated. Using the relation $R \approx (2\omega_rT)^2$ (see also footnote 9 of the paper), the factor reduces to $16/R$. Picking a reasonable R at resonance of 30 db, the second term will be smaller than the first by a factor of 0.016 and thus can be dropped. Thus, the important thing is not the accuracy of (9). Eq. (9) is used merely to obtain (27). Dropping $4\omega_r^2T^2$ in (9) has very little influence on (27). Eq. (27) can also be checked by showing that the values of the constants obtained in fitting (27) to the data are not way out of line as far magnitude. Perhaps it should be pointed out here that the caption under Fig. 2 may be misleading. All the values of the constants are normalized to magnitude of C_2 except C_6 so that it would be better to write them as $C_1/C_2 = 7.68 \times 10^{-24}$ sec^2 etc. The first constant C_1/C_2 can be written in terms of physical constants from (3), (4), (5), (18), and (19) of the paper.

$$\frac{C_1}{C_2} = \frac{\epsilon_0\mu_0}{\left(\frac{\pi}{a}\right)^2\left(\cot^2\frac{\pi x_0}{a} - 1\right)} \quad (1)$$

where from Fig. 1 of the paper $a = 0.148''$ $= 3.76 \times 10^{-3}$ m, and $x_0 = 2.27 \times 10^{-2}$ inches (measured from guide wall to center of ferrite slab). Putting in these values,

$$\frac{C_1}{C_2} = 5.95 \times 10^{-24} \text{ sec}^2,$$

which compares favorably with the fitted value of 7.68×10^{-24} sec^2. The other "constants" (C_3, C_4, C_5) contain parameters such as T, N_x, N_y, N_z, M and H_a which are not accurately known for hexagonal ferrites at millimeter-wavelengths so that meaningful theoretical values can not be obtained. So, even though many approximations were used in deriving (27), the authors feel that this equation is justified.

Bady has stated that line width seems to depend on the ferrite slab thickness. Eq. (38) which describes line width as a function of resonant frequency also depends on ferrite slab dimensions and slab positioning in the waveguide since the "constants" C_1 through C_5 are functions of these parameters. Therefore, (38) should not be so quickly discounted as a valid relation for hexagonal ferrites especially since the equation was able to approximate the experimental data with only one constant C_7 adjusted to fit the data.

In conclusion, the authors feel that the reason for Bady's misunderstanding is over approximation used on his derivation of equation of line width. The authors' procedure in deriving (38) and Bady's procedure in deriving (39) are both based on (7), but Bady dropped many terms which were supposed to be kept and derived (39) which is a well-known formula for line width of unbounded ferrite.[5] As the authors showed in this communication, (39) failed to explain the experimental results. If (39) could explain the experimental results, the authors would have never started this research. It is incorrect to apply a formula for unbounded ferrite to a bounded ferrite. It should be pointed out in this opportunity that the error of the extra 2 in the first term of (17) was not caught in reading the galley proofs.

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I. Bady[7]

This writer continues to disagree with the points of view expressed by Vilmur and Ishii in their reply. However, for the sake of brevity only two short comments will be made.

1) Vilmur and Ishii state in their reply that R, the ratio of reverse to forward attenuation of their single crystal barium fer-

[5] R. F. Soohoo, "Theory and Application of Ferrite," Prentice-Hall, Inc., Englewood Cliffs, N. J.; 1960.

[6] B. Lax and K. J. Button, "Microwave Ferrites and Ferrimagnetics" McGraw-Hill Book Co., Inc., New York, N. Y.; 1962.

[7] Received April 19, 1963.

rite isolator, was only 30 db. They show that if my formula for line width is used, this will result in a calculated line width of 2630 oersteds. Since this is an extraordinarily large line width for a single crystal, they claim that this proves my formula is incorrect. I say otherwise; if the line width of the single crystal is as small as claimed by the authors, how come the measured value of R is so extraordinarily low! In my opinion, this very low value of R proves that the sample thickness is so large that the RF fields are greatly distorted, and perturbation theory is not applicable.

2) Vilmur and Ishii claim that T, N_x, N_y, N_z, M, and H_a, are not known at *millimeter-wavelengths*. It is true that T is frequency and structure sensitive and must be determined experimentally. However, N_x, N_y, and N_z are a function of geometry only, and can be readily calculated from the graphs given in Figs. 4–6 of Lax and Button[6] in the authors' comments. M and H_a are basic material properties independent of frequency, and are given by Smit and Wijn[8] as 380 gauss and 17,000 oersteds, respectively.

P. Vilmur and K. Ishii[9]

1) Bady states in his rebuttal that R, the ratio of reverse to forward attenuation 30 db of single crystal barrium ferrite is "extraordinarily low"; and he claims that this low value of R is due to the large sample thickness. Bady did not give any theoretical reason why the large thickness made R low.

2) For the value of R, 30 db is a conservative value. However, is this "so extraordinarily low"? It is known that the single crystal barrium ferrite isolator's reverse attenuation is high, but the reverse to forward attenuation ratio is not necessarily high. More exact values of R are measured by using DBB attenuators[10] with an attenuation range of 34 to 37.5 db.

3) Bady disagrees with the narrow line width the author measured, but he did not state what the reasonable value of ΔH was. He did not give any theoretical support for his disagreement. Based on the measured value $\Delta H = 8.5$ oersteds, if Bady were correct, the bandwidth would be

$$\Delta\omega = \gamma \Delta H = 2\pi \times 238 \text{ Mc.}$$

which is

$$\frac{238 \text{ Mc}}{60 \text{ Mc}} \times 100\% = 396\%$$

off from the measured bandwidth which is impossible. If Bady were right,

$$T = \frac{2}{\Delta\omega} = 1.337 \times 10^{-9} \text{ sec.}$$

Then, using Lax's relation,

$$R = 4\omega_r^2 T^2 = 53.88 \text{ db}$$

[8] J. Smit and H. P. J. Wijn, "Ferrites," John Wiley and Sons, Inc. New York, N. Y., p. 204; 1959.

[9] Received May 13, 1963.

[10] K. Ishii, J. B. Y. Tsui, and F. F. Y. Wang, "Millimeter-wave field displacement type isolators with short ferrite strips," PROC. IRE, vol. 49, pp. 975–976; May, 1961.

which is far away from the measured value of 35.5 db. As shown above, it is impossible to explain the experimental results using Bady's simplified relation. From this numerical example, if R is assumed large, ΔH will be narrow but $\Delta\omega$ is too wide. If R is assumed too large, $\Delta\omega$ would be reasonable, but ΔH will be unreasonably narrow. Therefore, it is obvious that Bady's simplified relation is not usable in this case. It might be possible that a much higher value of backward-to-forward attenuation ratio could have been obtained by moving the sample closer to the center of the waveguide. But, the forward loss would have been excessive for a practical isolator. It is possible to consider that the ferrite supporting structure, and imperfections in the ferrite surface prevent the theoretical high forward-to-reverse attenuation ratio from being reached, if such a high ratio is possible. Note that, in the authors' analysis, these parameters were taken into consideration. All parameters are treated as the "equivalent quantities" rather than using the information obtained under idealized conditions.

4) Bady asks in his rebuttal that "if the line width of the single crystal is as small as claimed by the authors, how come the measured value of R is so extraordinarily low." The reason was already explained by the authors' complicated equations given in the original paper, and by numerical examples given in the authors' previous reply and in this letter. On the contrary, Bady's simple equations failed to fit the experimental data as has been shown.

5) In the original paper, the authors admit that the authors' complicated equations are *not exact* equations but for many reasons they are approximations. One of the reasons may be the perturbation method as Bady pointed out. In a strict sense, the perturbation theory may *not* be applicable to obtain *exact* quantities. The authors showed that, in the original paper, the perturbation theory is still applicable to obtain *approximate quantities within the experimental uncertainties.*

6) Concerning the second point made by Bady; he may have a valid complaint about the authors' statement that N_x, N_y, N_z, etc. are not known at *millimeter wavelengths* because this statement implies that these parameters are functions of frequency. Parameters N_x, N_y, N_z, M, and H_a were originally defined under the magnetostatic fields.[6,8] In such a case, as Bady stated, "N_x, N_y, N_z are functions of geometry only, and M and H_a are basic material properties." In microwaves, specifically the case of the isolator analysis, Lax[4] assumed

$$h_{i\substack{x\\y\\z}} = h_{0\substack{x\\y\\z}} - N_{\substack{x\\y\\z}} M_{\substack{x\\y\\z}}. \qquad (1)$$

Now, in this case, h_i, h_0 and M are RF quantities and therefore N_x, N_y, N_z, and M are not any more exactly equal to the quantities defined in the magnetostatic fields.

These quantities, specifically M, generally depend on the direction of magnetization. In the magnetostatic case N_x, N_y, and N_z are defined assuming:

(a) The ferrite sample is uniform material.
(b) The surface is perfectly smooth.
(c) The damping parameter $\alpha = 1/\omega T$ has nothing to do with an opposing internal field due to the alignment of all spins with the applied magnetostatic field. All spins are always aligned in the direction of the magnetostatic field.
(d) The magnetic field distribution inside the material is uniform except for discontinuities at the boundaries.
(e) The ferrite sample is completely isolated.
(f) The ferrite sample has a shape of an equivalent ellipsoid.

On the contrary, in the case of the millimeter-wave ferrite isolators which the authors have discussed:

(a) The ferrite sample material was fairly uniform when checked by eyes, but some thin cracks were found.
(b) The surface was fairly smooth except for those cracks.
(c) The effect of the damping parameter must be taken into consideration. As shown in the authors' previous letter, T is on the order of 10^{-11} sec in millimeter wavelengths which is comparable to the period of millimeter waves which is on the order of 10^{-11} sec. As Bady admits, T is a function of operating frequency.
(d) In millimeter waves, since the wave length is short and the period of the RF signal is close to the phenomenological relaxation time, and because of the "skin effect" and local resonance, a uniform distribution of RF magnetic field inside the sample is not likely. Also, the field distribution pattern would be a function of operating frequency.
(e) The ferrite sample was not isolated. The ferrite sample was glued on a polystyrene slab and mounted in the waveguide. The effect of the presence of the polystyrene and glue dielectrics upon the RF field distribution around the sample should not be omitted when defining the demagnetizing factors by (1). These things are all frequency dependent, and the ferrite sample was coupled to the waveguide field distribution which was also frequency dependent.
(f) The equivalent ellipsoid concept was proved to be true in the magnetostatic field,[11] but to the authors' knowledge, no evidence of validity was found in the literature for millimeter-wavelengths on the order of 3 mm.

These are the reasons why the authors said in the previous letter that N_x, N_y, N_z are not *accurately* known at millimeter-wavelengths.

7) Concerning the saturation magnetization M in the isolator analysis, Lax[6,4] defined it as the RF quantity instead of magnetostatic quantity as shown in (1). Therefore, M is now dependent on the phenomenological relaxation time T of the spin alignment[6]

[11] J. A. Osborn, "Demagnetizing factors of the general ellipsoid," *Phys. Rev.*, vol. 67, p. 351; 1945.

which is comparable to the operating RF period. As Bady admits in his rebuttal, "T is frequency and structure sensitive and must be determined experimentally." Therefore, M is frequency dependent at millimeter-wavelengths. Then the anistrophy field[8]

$$H_a = \frac{-2(K_1 + 2K_2)}{M} \text{ or } \frac{2K_1}{M} \quad (2)$$

(depending on the direction of the anisotrophy field) is also frequency dependent because M appears in (2). Using Kittel's relation,

$$\omega_r^2 = [\omega_0+(N_x-N_z)\omega_M][\omega_0+(N_y-N_z)\omega_M]$$
$$= \gamma^2[H_0+H_a]^2 \quad (3)$$
$$H_a = \frac{\sqrt{[\omega_0+(N_x-N_z)\omega_M][\omega_0+(N_y-N_z)\omega_M]}}{\gamma} - H_0 \quad (4)$$

the frequency dependence of H_a is also obvious because (4) contains N_x, N_y, N_z and $\omega_M = 4\pi\gamma M$. For these reasons, the authors stated in the previous letter that M and H_a are not *accurately* known at millimeter-wavelengths.

8) In microwaves, it is possible to consider that, at a frequency which is low enough so that the phenomenalogical relaxation time T is small compared with the period of the operating microwave frequency, the quantity found by the magnetostatic method may be usable. In fact, Kittel,[12] and J. Smit and H. G. Beljers,[13] showed experimentally that the quantity found in the static method is applicable in the range of microwave frequencies.

9) The values of M, 380 gauss and H_a, 17000 oersted given by Smit and Wijn[8] were obtained by the static method in a specified direction of magnetization and not at millimeter wavelengths. Validity of these values at millimeter-wavelengths in the range of 3 mm is questionable for the reasons mentioned above. The application of these quantities to the authors' complicated isolators is even more questionable. The value of H_a at the 5 mm wave-length range[14] was estimated to be 18,400 oersted for a single crystal of $BaFe_{12}O_{19}$ of density 5.13 g/cm^3 which is 97 per cent of the true X-ray density.

The authors sincerely appreciate Bady's interesting questions.

I. Bady[15]

This writer continues to disagree with many statements made by Vilmur and Ishii, but for the sake of brevity will make only one short comment. Vilmur and Ishii make several remarks such as, "Bady does not give any theoretical reason why the large thickness made R low," ". . . it is impossible to explain experimental results using Bady's simplified relations." I have clearly stated in my rebuttal that the large thickness of the sample greatly distorts the RF fields in the sample (as compared to what the fields would be in an empty waveguide) and this makes perturbation theory inapplicable. Hence R cannot be calculated by Lax's formula, as done by Vilmur and Ishii, since the formula is based on perturbation theory. Also I have clearly stated in my original comments that I am not surprised that the simple formula for linewidth does not fit experimental data, since the large sample thickness makes perturbation theory inapplicable and anomolous results may occur.

P. Vilmur and K. Ishii[16]

1) Bady's explanations are qualitative in nature. What the authors want is an exact quantitative theoretical proof to support Bady's conclusion.

2) Inapplicability of the perturbation theory may not guarantee low value of R.

3) Bady states that the large sample makes his formulas inapplicable. This implies that if the sample is made smaller, the sample will follow Bady's simplified formula. Here is a problem to be cleared in Bady's approach. If the sample is made smaller, the sample will follow more exactly Lax's formula instead of Bady's simplified formula, because, as Bady has been asserting, the perturbation theory is applicable with less errors for smaller samples.

[12] C. Kittel, "Interpretation of anomalous Larmor frequencies in ferromagnetic resonance experiment," *Phys. Rev.*, vol. 71, pp. 270–271; February, 1947.

[13] J. Smit and H. G. Beljers, "Ferromagnetic resonance absorption in $BaFe_{12}O_{19}$, a highly anisotropic crystal," *Philips Res. Rept.*, vol. 10, pp. 113–130; 1955.

[14] F. F. Y. Wang, K. Ishii, J. B. Y. Tsui "Ferrimagnetic resonance of single-crystal barium ferrite in the millimeter-wave region," *J. Appl. Phys.*, vof. 32, pp. 1621–1622; August, 1961.

[15] Received May 21, 1963.

[16] Received June 13, 1963.

Ferromagnetic Line Width of Nonoriented Polycrystalline Hexagonal Ferrites with Large Magnetic Anisotropy Fields*

Introduction

Data on the line widths of oriented polycrystalline, hexagonal ferrites with large magnetic anisotropy fields have shown that the uniaxial ferrites (easy direction of magnetization along the C axis) have a considerably larger line width than that of planar ferrites (easy plane of magnetization perpendicular to the C axis). For example, in work performed at Philips[1] on the uniaxial barium and strontium ferrites of the magnetoplumbite structure, with aluminum or titanium-cobalt substitutions, the line width varied over a range of 1600 to 3300 oersteds for materials with anisotropies ranging from 7000 to 52,000 oersteds. There was no strong correlation between line width and anisotropy field. In work done at Sperry[2] on uniaxial nickel-W compounds with cobalt substitutions, the line width ranged from 2200 to 3000 oersteds for materials with anisotropies ranging from 7000 to 12,800 oersteds. On the other hand, in work performed by RCA on planar ferrites, a line width as low as 110 oersteds was obtained,[3] and a large number of compounds had a line width less than 500 oersteds.[4]

It is very unlikely that the large line width of polycrystalline uniaxial ferrites is due to the crystallite's line width Though relatively little work has been done on single crystals of hexagonal ferrites, a line width of 50 oersteds has been obtained on a single crystal of barium ferrite[5] and on a single crystal of aluminum substituted strontium ferrite.[6] A line width of 18 oersteds was obtained on a single crystal of the planar ferrite Zn_2Y.[7] However, there has been considerably more research done on single crystals of Zn_2Y ferrite than on those of uniaxial ferrites, to reduce line width.

A major contribution to the line width of oriented hexagonal ferrites, both of uniaxial and planar types, was considered to be imperfect orientation. It was therefore desirable to study the extreme case of imperfect orientation, *i.e.*, completely nonoriented materials, and compare the theoretically calculated line widths of the uniaxial and planar ferrites for this case.

Method of Calculation

Only a brief outline of the method used to calculate the line widths of the nonoriented uniaxial and planar ferrites will be given in this communication. More details are contained in a Technical Report[8] with the same title published by the United States Army Electronic Research and Development Laboratory.

The nonoriented ferrite was assumed to be composed of small, single domain crystallites whose C axes were randomly oriented over all possible solid angles. It was further assumed that the crystallites

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[1] D. J. DeBitetto, F. K. duPré, and F. G. Brockman, "Hexagonal Magnetic Materials for Microwave Applications," Philips Laboratories, Irvington-on-Hudson, N. Y., Final Rept , Contract DA36-039 SC-85279; July, 1961.

[2] G. Rodrique and J. Pippin, "Theoretical and Experimental Investigation to Determine the Microwave Characteristics and Applications of Hexagonal Magnetic Oxides to Microwave Circuitry," Sperry Microwave Electronics Co., Clearwater, Fla., Tech. Rept. Contract AF30 (602) 2330; December, 1961.

[3] R. Harvey, I. Gordon, and R. Braden, "Hexagonal Magnetic Compounds," RCA Laboratories, Princeton, N. J. Quarterly Rept. No. 6, Contract DA36-039 SC-87433; December, 1962.

[4] R. Harvey, I. Gordon, and R. Braden, "Hexagonal Magnetic Compounds," RCA Laboratories, Princeton, N. J , Final Rept., Contract DA36-039 SC-78288; June, 1961.

[5] I. Bady, T. Collins, D. J. DeBitetto, and F. K. duPré, "Ferromagnetic linewidth of single crystals of barium ferrite ($BaFe_{12}O_{19}$)," Proc. IRE (*Correspondence*), vol. 58, p. 2033; December, 1960.

[6] D. J. DeBitetto, F. K. duPré, and F. G. Brockman, "Hexagonal Magnetic Materials for Microwave Applications," Philips Laboratories, Irvington-on-Hudson, N. Y., Final Rept., Contract DA36-039 SC-78071; July, 1961.

[7] A. Tauber, R. Savage, R. Gambino, and C. Whinfrey, "Growth of single crystal hexagonal ferrites containing Zn," *J. Appl. Phys.*, Suppl. to vol. 33, pp. 1381–1382, March, 1962.

[8] I. Bady and G. McCall, "Linewidth of nonoriented polycrystalline hexagonal ferrites with large magnetic anisotropy fields," U. S. Army Electronic Research and Development Laboratory, Fort Monmouth, N. J., Technical Rept. 2350,; March, 1963.